



K22U 0414

Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS – OBE – Regular)

Examination, April 2022

(2019 Admission)

CORE COURSE IN MATHEMATICS

6B11 MAT : Complex Analysis

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the real and imaginary parts of the function $f(z) = \frac{1}{z}$.

2. Evaluate $\int_0^{1+i} z^2 dz$.

3. State Morera's theorem.

4. Write the Laurent series for $z^2 e^{\frac{1}{z}}$.

5. Find residue of $f(z) = \frac{\sin z}{z^4}$.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Solve $\cos z = 5$.

7. Find the Principal value of $\ln(i)$.

8. Evaluate $\int_C \operatorname{Re}(z) dz$, where $C : z(t) = t + 2it$, $(0 \leq t \leq 1)$.

9. Show that the fundamental region of e^z is $-\pi < y \leq \pi$.

10. Find an upperbound for the absolute value of $\int_C z^2 dz$.

11. State identity theorem for power series.
12. Define absolute convergence and conditional convergence.
13. Check the convergence of $\sum_{n=0}^{\infty} \frac{(100 + 75i)^n}{n!}$.
14. Show that sequence $\{z_n = x_n + iy_n\}$ converges to $c = a + ib$ if and only if $\{x_n\}$ converges to a and $\{y_n\}$ converges to b .
15. State Picard's theorem.
16. $\int_C \frac{z^3 - 6}{2z - i} dz$ where C is $|z| = \frac{3}{4}$.

PART - C

Answer **any four** questions. **Each** question carries **four** marks.

17. Verify $u = x^2 - y^2 - y$ is harmonic and find the harmonic conjugate of u .
18. Find $(1 + i)^{2-i}$.
19. State and prove Cauchy's inequality.
20. State and prove Liouville's Theorem.
21. Find radius of convergence of the following.

a) $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z - 3i)^n$.

b) $\left[(-1)^n + \frac{1}{2^n} \right] z^n$.

22. Find residue at poles of the function $f(z) = \frac{9z + i}{z^3 + z}$.

23. Classify isolated singularities. Give suitable examples too.

PART - D

Answer any two questions. Each question carries six marks.

24. a) State and prove necessary condition for differentiability.
b) If f is an analytic function with $|f|$ constant, then show that f is constant.
25. a) State Cauchy's Integral Formula.
b) $\int_C \frac{z^2 + 1}{z^2 - 1} dz$, where C is $|z - 1| = 1$.
c) $\int_C \frac{\tan z}{z^2 - 1} dz$, where C is $|z - \pi/2| = \frac{1}{4}$.
26. a) Find Maclaurin's series for $f(z) = \frac{1}{(1+z)^2}$.
b) Find Taylor's series for $f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$.
27. a) State and prove Cauchy Residue Theorem.
b) Evaluate $\int_C \left(\frac{ze^{\pi z}}{z^4 - 16} + ze^{\frac{\pi}{z}} \right) dz$, where C is the ellipse $9x^2 + y^2 = 9$.